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A RUDIMENTARY THEORY OF TOPOLOGICAL FOUR-DIMENSIONAL GRAVITY

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ABSTRACT. A theory of topological gravity is a homotopy-theoretic representation of the Segal-Tillmann topologification of a two-category with cobordisms as morphisms. This note describes a relatively accessible example of such a thing, suggested by the wall-crossing formulas of Donaldson theory.

1. GRAVITY CATEGORIES

A **cobordism category** has manifolds as objects, and cobordisms as morphisms. Such categories were introduced by Milnor [14], but following Segal's definition of conformal field theory [23] and Atiyah's subsequent abstraction of the notion of topological quantum field theory [1] they have been studied very widely. Recently, Tillmann [25] has demonstrated the utility of certain closely related **two-categories**; the definition below is based on her ideas.

Definition A **gravity two-category** has

- (closed) **manifolds** as objects,
- **cobordisms** as morphisms, and
- **isomorphisms** of these cobordisms, equal to the identity on the boundary, as **two-morphisms**.

There are many possible variations on this theme, and I will not try for maximal generality. If the objects of the category have dimension d (so the cobordisms are $(d+1)$ -dimensional) then I will say that the gravity (two-)category is $(d+1)$ -dimensional. I will assume that manifolds are smooth, compact and oriented, but not necessarily connected, and (following Segal) I understand the empty set to be a manifold of any dimension.

1.1 If V and V' are d -manifolds, a morphism

$$W : V \rightarrow V'$$

is (the germ of) an orientation-preserving diffeomorphism

$$(V_{op} \cup V') \times [0, 1] \cong \nu(\partial W)$$

of the manifold on the left with a collar neighborhood of the boundary of the $(d+1)$ -manifold W ; the subscript *op* signifies reversed orientation. The morphism category $Mor(V, V')$ has such cobordisms as its objects; it is a topological category, in which

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the space of morphisms between two cobordisms W and \tilde{W} consists of orientation- and boundary-identification-preserving diffeomorphisms $W \cong \tilde{W}$. Gluing along the boundary defines a continuous composition functor

$$W, W' \mapsto W \circ W' : \text{Mor}(V, V') \times \text{Mor}(V', V'') \rightarrow \text{Mor}(V, V'') ,$$

while disjoint union of objects gives this two-category a monoidal structure, with the empty set as identity object.

By replacing $\text{Mor}(V, V')$ with its set $\pi_0 \text{Mor}(V, V')$ of equivalence classes of objects, we obtain the category employed by Atiyah to define a topological quantum field theory; in other words, we can pass from a gravity two-category, in which the morphism objects are enriched by a categorical structure, to a classical category, in which the morphism objects are simply sets. Tillmann's more perspicacious alternative is to interpret the topological category $\text{Mor}(V, V')$ as a simplicial topological space and to replace it with its geometric realization $\text{Mor}(V, V')$. This construction preserves Cartesian products (as does π_0 : indeed the set of equivalence classes of objects in Mor is the set of components of the space Mor), defining a **topological gravity category** (i.e., a category in which the morphism objects are topological spaces, and the composition maps are continuous). A topological quantum field theory in the sense of Atiyah is thus a (continuous) monoidal functor from a topological gravity category to the (topological) category of modules over a **discrete** topological ring.

However, we can consider monoidal functors to more general categories: for example, the singular chains on the morphism spaces of a gravity category define a monoidal category enriched over chain complexes, whose representations are the (co)homological field theories [12] of physics. In the language of homotopy theory, these are representations in a category of modules over some Eilenberg-MacLane ring-spectrum. In general, I will call any monoidal functor from a topological gravity category to the category of dualizable objects over a ring-spectrum, a **theory of topological gravity**. This paper is concerned with some rather straightforward examples of theories of four-dimensional topological gravity, motivated by the wall-crossing formulas of Donaldson theory.

1.2 The terminology needs explanation. If W is a manifold with boundary, let $\text{Diff}_+(W)$ be the topological group of orientation-preserving diffeomorphisms of W which restrict to the identity in some neighborhood of ∂W . The components of $\text{Mor}(V, V')$ are indexed by equivalence classes of cobordisms $W : V \rightarrow V'$, and the components themselves are the classifying spaces $B\text{Diff}_+(W)$. Gluing [13] defines a continuous homomorphism

$$\text{Diff}_+(W) \times \text{Diff}_+(W') \rightarrow \text{Diff}_+(W \circ W') ;$$

thus the (components of the) composition map in the topological gravity category are the maps these compositions induce on classifying spaces.

On the other hand, a fundamental tautology of Riemannian geometry asserts that an isometry of a complete connected Riemannian manifold which fixes a frame at some point is the identity: such a map preserves the geodesics out of the framed point, and any other point in the manifold can be reached by such a geodesic. It follows that group of diffeomorphisms framing some basepoint will act **freely** on

the (contractible) space of Riemannian metrics on a compact connected manifold. The space $B\text{Diff}_+$ is the homotopy quotient of the space of metrics [7] by the diffeomorphism group and we can think of morphisms in the $(d+1)$ -dimensional gravity category as cobordisms between d -manifolds, together with a choice of equivalence class of Riemannian metric on the cobordism.

A (projective) Hilbert-space representation of a topological gravity category, along the lines considered by Segal in his definition of a conformal field theory, is thus very close to a quantum theory of gravity. When $d = 1$ we can see this more explicitly: the Riemann moduli space is the quotient of the space of conformal structures on a closed connected surface by the group of its orientation-preserving diffeomorphisms, which acts with finite isotropy when the genus exceeds one. This defines a monoidal functor from the two-dimensional gravity category to Segal's, which (away from closed surfaces of low genus) is a rational homology isomorphism on morphism spaces. Consequently, any conformal field theory in Segal's sense defines a quantum theory of two-dimensional gravity.

1.3 Examples:

- i) There is no *a priori* reason to limit ourselves to smooth manifolds: we can begin with a two-category of topological or piecewise-linear manifolds, and replace its morphism categories by their classifying spaces, as before: there are lots of non-smoothable four-manifolds!
- ii) In higher dimensions, the category of manifolds and equivalence classes of s -cobordisms is a groupoid, with the Whitehead group of an object as its automorphisms. In low dimensions these categories are quite mysterious.
- iii) We can consider classes of manifolds with extra structure: by assuming that the second Stiefel-Whitney class is zero, we can define a gravity category of four-dimensional Spin-manifolds. [The set of Spin-structures on such a manifold is a principal homogeneous space over its first mod two cohomology group, but is not naturally isomorphic to that group.]
- iv) Similarly, the four-dimensional gravity category of $\text{Spin}^{\mathbb{C}}$ -manifolds is obtained from manifolds and complex line bundles over them, with Chern class lifting w_2 .

Any smooth four-manifold admits a $\text{Spin}^{\mathbb{C}}$ -structure, so example iv) contains example iii) as a subcategory. Note that the Chern class of a complex line bundle on a smooth closed connected four-manifold which lifts w_2 has square equal to $2\chi + 3\sigma$. This abstracts a classical property of the canonical bundle on a complex algebraic surface.

When d is odd, the morphisms of a $d+1$ -dimensional gravity category are naturally graded by Euler characteristic: the correction term in the formula

$$\chi(W \circ W') = \chi(W) + \chi(W') - \chi(W \cap W')$$

is zero. When d is one, the Euler characteristic counts the number of handles or loops in the usual quantum or genus expansion; it defines a zeroth Mumford class κ_0 . If we exclude closed manifolds from our morphism spaces, and thus do not admit the empty set as a plausible object, this grading is bounded below.

Many decorations of gravity categories are possible: Lorentz cobordism [22,26], defined by a nowhere-vanishing vector field oriented suitably at the boundary, is one interesting example. Restricting the object manifolds (e.g. to be unions of homology spheres, or contact manifolds [11]) is another alternative. Witten's original two-dimensional theory [27] admits singular (stable) algebraic curves as morphisms; this compactifies its morphism spaces, and Kontsevich has shown (as Witten conjectured) that the resulting theory has a well-behaved vacuum state.

2. PRETTY GOOD TOPOLOGICAL GRAVITY

A Riemannian metric g on an oriented closed connected two-manifold Σ defines a Hodge operator $*_g$ on its harmonic forms. This operator squares to -1 on one-forms, and so defines a complex structure on the de Rham cohomology $H_{dR}^1(\Sigma)$. The space of isomorphism classes of complex structures on a real Euclidean space of dimension $2g$ is the quotient $SO(2g)/U(g)$, so we get a map

$$\tau : B\text{Diff}_+(\Sigma) \rightarrow (\text{Met})/(\text{Diff}_+) \rightarrow SO/U$$

in the large genus limit. This can be constructed more generally by working with differential forms which vanish on the boundary. Orthogonal sum of vector spaces makes an H -space of the target of τ , and it is not hard to see that if Σ and Σ' are surfaces with geodesic boundaries, then gluing them c times along some sets of compatible boundary components defines a homotopy-commutative diagram

$$\begin{array}{ccc} B\text{Diff}_+(\Sigma) \times B\text{Diff}_+(\Sigma') & \longrightarrow & B\text{Diff}_+(\Sigma \circ \Sigma') \\ \downarrow \tau \times \tau & & \downarrow \tau \\ SO/U \times SO/U & \xrightarrow{\oplus} & SO/U \end{array}$$

[The intersection form on the middle homology of $\Sigma \circ \Sigma'$ is the direct sum of the intersection forms of Σ and Σ' , together with a **split hyperbolic** intersection form of rank $c - 1$, which has a canonical complex structure.]

2.1 This is perhaps the simplest example of a theory of two-dimensional topological gravity: it is a monoidal homotopy-functor to a topological category SO/U with one object and the H -space SO/U of morphisms [18]. The functor is actually quite classical: it is a version of the Jacobian, which refines the infinite symmetric product construction. [The Siegel moduli space for abelian varieties has the rational cohomology of an integral symplectic group which, by a version of the Hirzebruch proportionality principle, has the stable rational cohomology of SO/U .]

The objects of the two-dimensional gravity category are just collections of circles, which are indexed by integers. In this situation, a theory of topological gravity with values in the category of k -module spectra is defined by a dualizable k -module spectrum M , together with a system of characteristic classes

$$\tau_q^p \in (\bar{M}^{\wedge p} \wedge_k M^{\wedge q})^*(B\text{Diff}_+\Sigma)$$

for bundles of connected surfaces Σ with p incoming and q outgoing boundary components, which behave compatibly under gluing. [Here $M^{\wedge q}$ is the q -fold smash

(or tensor) product of copies of M , over k , \bar{M} is the k -dual of M , and gluing is to be compatible with the composition operation defined by the trace map

$$\bar{M} \wedge_k M \rightarrow k.$$

The example above is deceptively simple, for in this case $M = k$. In more general cases, related to quantum cohomology, M will be a Frobenius k -algebra [17].

2.2 This Hodge-theoretic construction has a close analogue for four-manifolds, which is also classical in a way: it is a descendant of the wall-crossing formulas [19] of Donaldson theory. As in the two-dimensional example, it uses basic properties of the intersection form on middle cohomology:

If W is an compact connected oriented four-manifold with ∂W a union of homology spheres then the intersection form

$$x, y \mapsto \langle x, y \rangle = (x \cup y)[W, \partial W]$$

on the integral lattice $B = H^2(W, \partial W, \mathbb{Z})$ is unimodular. In dimension four, Wu's formula implies that

$$q(x) = \langle x, x \rangle \equiv \langle x, w_2 \rangle$$

modulo two, so the form q is even iff the manifold admits a spin-structure. If, more generally, the manifold has a $Spin^C$ -structure, then the intersection form is even or odd depending on the parity of the Chern class of its associated complex line bundle.

By a fundamental theorem of Freedman [8] any unimodular quadratic form can arise as the intersection form of a closed topological four-manifold; but by equally fundamental results of Donaldson [6] the intersection form of a closed **smooth** four-manifold is either indefinite, or diagonalizable over the integers.

As in two dimensions, the action of a diffeomorphism on homology defines a monodromy representation

$$\text{Diff}_+(W) \rightarrow \text{Aut}_+(B, q) = \text{SO}(B)$$

which factors through $\pi_0(\text{Diff}_+(W))$; it is convenient to think of its kernel [10] as an analogue, for four-manifolds, of the Torelli group of surface theory.

2.3 Let b be the rank, and $\sigma = b_+ - b_-$ the signature, of the inner product space defined by q on $B \otimes \mathbb{R}$. We will be most interested in **indefinite** lattices: these are classified by their rank, signature, and type (even if $q(x) \equiv 0 \pmod{2}$, otherwise odd). In the indefinite case, the manifold $\text{Grass}^-(B)$ of maximal negative-definite subspaces of $B \otimes \mathbb{R}$ is a noncompact (contractible) symmetric space defined by a cell of dimension $b_+ b_-$ in the usual Grassmannian of b_- -planes in b -space. The orthogonal group of the lattice acts on this cell with finite isotropy, so the canonical homotopy-to-geometric quotient map

$$BSO(B) \rightarrow \text{Grass}^-(B)/SO(B)$$

is a rational homology isomorphism. If B and B' are indefinite lattices, then the map which sends a pair of negative definite subspaces in the real span of each, to their orthogonal sum in the real span of the direct sum lattice, defines a map

$$\text{Grass}^-(B) \times \text{Grass}^-(B') \rightarrow \text{Grass}^-(B \oplus B')$$

which is equivariant with respect to the Whitney sum homomorphism

$$\mathrm{SO}(B) \times \mathrm{SO}(B') \rightarrow \mathrm{SO}(B \oplus B')$$

The Grothendieck group of the category of indefinite even unimodular lattices is free abelian on two generators, corresponding to the hyperbolic plane and the E_8 lattice [24 Ch. V]. The ‘Hasse-Minkowski’ spectrum $\mathrm{HMK}(\mathbb{Z})$ defined by the algebraic K -theory of the category of such lattices is the group completion of the monoid constructed from the disjoint union of the classifying spaces of their orthogonal groups; the tensor product of two such lattices defines another, so this is actually a commutative ring-spectrum.

2.4 A Riemannian metric g on W defines a Hodge operator $*_g$ on harmonic forms, but now this operator squares to $+1$ on the middle cohomology. The function which assigns to g , the $*_g = -1$ -eigenspace of harmonic two-forms vanishing on ∂W , maps the space of Riemannian metrics to the negative-definite Grassmannian $\mathrm{Grass}^-(B)$. This map is equivariant with respect to the action of $\mathrm{Diff}_+(W)$.

If W and W' are four-manifolds bounded (as above) by homology spheres, and if $W \circ W'$ results from gluing these manifolds along a collection of compatible boundary components, then the quadratic module of $W \circ W'$ is canonically isomorphic to $B \oplus B'$; hence the cohomology representation of the diffeomorphism group defines a monoidal functor from the gravity category of spin four-manifolds bounded by homology spheres, to the topological category **HMK** with one object, and the Hasse-Minkowski spectrum as morphisms.

3. TOWARD A PARAMETRIZED DONALDSON THEORY

A good theory of gravity shouldn’t exist in a vacuum: it deserves to be coupled to some nontrivial matter. Donaldson [5] and Moore and Witten [16] have suggested the study of an ‘equivariant’ Yang-Mills theory parameterized by classifying spaces of diffeomorphism groups. A fragment of such a theory is sketched below.

3.1 Suppose W is closed and, for simplicity, connected and simply-connected. The graded space $\mathrm{Bun}_*(W)$ of gauge equivalence classes of connections on $\mathrm{SU}(2)$ -bundles over W has components indexed by the second Chern class of the bundle. Let \mathbf{D}_* be the subspace of $\mathrm{Met} \times \mathrm{Bun}_*(W)$ consisting of pairs (g, A) , where A is a connection on an $\mathrm{SU}(2)$ -bundle over W with curvature two-form

$$*_g(F_A) = -F_A$$

antiselfdual with respect to the metric g . The standard transversality arguments of Donaldson theory [5 §4.3] imply that this space is a manifold, with fiber of dimension $8c_2 - 3(b_+ + 1)$ above the metric g ; at least, provided this metric admits no **reducible** antiselfdual connections. Such reducible connections define an interesting kind of distinguished boundary to the space of antiselfdual connections.

3.2 More precisely, the wall arrangement

$$\mathrm{Wall}(B) = \{H \in \mathrm{Grass}^-(B) \mid H \cap B \neq \{0\}\}$$

of the lattice B is the set of maximal negative-definite subspaces of $B \otimes \mathbb{R}$ containing a lattice point; it is a union of smooth submanifolds of codimension b_- . It is filtered

by the increasing family $\text{Wall}_d(B)$ of subspaces consisting of maximal negative-definite H containing a lattice point x with $0 > q(x) \geq -d$; this is a **locally finite** union of submanifolds [9]. The orthogonal group of B acts naturally on the wall arrangement, as well as on the quotients

$$\mathbf{X}_d(B) = \text{Grass}^-(B)/\text{Wall}_d(B)$$

(which are roughly S -dual to the wall arrangements). If B and B' are two indefinite lattices, then the orthogonal direct sum map defines a commutative diagram

$$\begin{array}{ccc} \text{Grass}^-(B) \times \text{Grass}^-(B') & \longrightarrow & \text{Grass}^-(B \oplus B') \\ \downarrow & & \downarrow \\ \mathbf{X}_d(B) \wedge \mathbf{X}_{d'}(B') & \longrightarrow & \mathbf{X}_{d+d'}(B \oplus B') \end{array}$$

which is equivariant, with respect to the Whitney sum on orthogonal groups.

3.3 If g is in the complement of the preimage Met_d^0 of Wall_d in the space Met of metrics on W , then no $\text{SU}(2)$ -bundle with Chern class less than $-d$ admits a connection with $*_g$ -antiselfdual curvature. Thus if \mathbf{D}_d^0 denotes the space of pairs (g, A) such that A is gauge equivalent to a connection induced from a line bundle with curvature antiselfdual with respect to g , then

$$(\mathbf{D}_d, \mathbf{D}_d^0) \rightarrow (\text{Met}, \text{Met}_d^0) \times \text{Bun}_d(W)$$

is a kind of $\text{Diff}_+(W)$ -equivariant cycle, of relative finite dimension above the space of metrics. It cannot be expected to be proper, but Donaldson theory has developed sophisticated methods to deal with such issues [4]: let $\text{SP}_d^\infty(W_+)$ be the space of finitely supported functions f from W to the integers, such that

$$\sum_{x \in X} f(x) = d,$$

and let

$$\overline{\mathbf{D}}_d = \coprod_{0 \leq i \leq d} \mathbf{D}_i \times \text{SP}_{d-i}^\infty(X_+)$$

be the analogue of the Uhlenbeck - Donaldson compactification of \mathbf{D}_d in the stratified space

$$\text{Met} \times \left(\coprod_{0 \leq i \leq d} \text{Bun}_i(W) \times \text{SP}_{d-i}^\infty(X_+) \right) = \text{Met} \times \overline{\text{Bun}}_d(W).$$

Completing the subspace \mathbf{D}_d^0 of reducible connections analogously defines a candidate

$$(\overline{\mathbf{D}}_d, \overline{\mathbf{D}}_d^0) \rightarrow (\text{Met}, \text{Met}_d^0) \times \overline{\text{Bun}}_d(W)$$

for a $\text{Diff}_+(W)$ -equivariant Donaldson cycle.

To extract homological information from this construction, note that a k -dimensional class z in the rational homology of $B\text{Diff}_+(W)$ maps to a sum, with rational coefficients, of homology classes defined by maps

$$Z \rightarrow \text{Met} \times_{\text{Diff}_+} \text{pt}$$

of **smooth** manifolds Z . Its fiber product with the projection

$$\overline{\mathbf{D}}_d \rightarrow \text{Met} \times_{\text{Diff}_+} \overline{\text{Bun}}_d(W) \rightarrow \text{Met} \times_{\text{Diff}_+} \text{pt}$$

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defines a class of dimension $k + 8d - 3(b_+ + 1)$ in the rational homology of

$$(\text{Met}, \text{Met}_d^0) \times_{\text{Diff}_+} \overline{\text{Bun}}_d(W).$$

3.4 The homotopy-to-geometric quotient map for the space of connections is a rational homology equivalence of $\text{Bun}_*(W)$ with the space of based smooth maps from W_+ to $BSU(2)$ [6 §5.1.15], and the Pontrjagin class defines another rational homology isomorphism with the space of maps to the Eilenberg - MacLane space $H(\mathbb{Z}, 4)$. By the Dold-Thom theorem,

$$\pi_i \text{Maps}(W_+, H(\mathbb{Z}, 4)) \cong H^{4-i}(W, \mathbb{Z}) \cong H_i(W, \mathbb{Z}) \cong \pi_i(\text{SP}^\infty(W_+))$$

so for many purposes we can replace the space of $SU(2)$ -connections by the free topological abelian group on W . [This identification uses Poincaré duality, and hence requires a choice of orientation: the space of bundles is a contravariant functor, but the infinite symmetric product is covariant.] Combined with the constructions outlined above, this defines a generalized Donaldson invariant as a homomorphism

$$\mathcal{D}_d : H_*(B\text{Diff}_+, \mathbb{Q}) \rightarrow H_{*+8d-3(b_++1)}(\mathbf{X}_d \wedge_{\text{SO}} \text{SP}_d^\infty, \mathbb{Q})$$

with values in a group which depends only on the cohomology lattice B ; indeed the rational homology of $\text{SP}^\infty(W_+)$ is the symmetric algebra on the homology of W , and the automorphic cohomology

$$H_{\text{SO}(B)}^*(\text{SP}^\infty(W_+), \mathbb{Q})$$

contains the classical ring of automorphic forms for the orthogonal group, as the invariant elements of the symmetric algebra on B .

This invariant generalizes the usual one, in the sense that \mathcal{D}_d on a degree zero generator of the homology of $B\text{Diff}_+$ is the classical invariant. [The usual convention is to interpret the antiselfdual cycle as a function on the cohomology of W , by taking its Kronecker product with $\exp(x)$, $x \in H^*(X)$.] A four-manifold is said to be of **simple** type, if the behavior of its classical invariant as a function of charge is not too complicated: in the present formalism, the condition is that

$$\mathcal{D}_{d+1}(1) \mapsto w_0 w_4^2 \mathcal{D}_d(1)$$

under the homomorphism induced by the restriction map from \mathbf{X}_{d+1} to \mathbf{X}_d (where w_0 and w_4 generate the homology in degrees zero and four of W). This suggests

$$\tilde{\mathcal{D}}_d = (w_0 w_4^2)^{-d} \mathcal{D}_d \in \text{Hom}^{-3(b_++1)}(H_*(B\text{Diff}_+), H_*(\mathbf{X}_d \wedge_{\text{SO}} \text{SP}_0^\infty))$$

as the natural normalization for the generalized invariant.

4. ON THE INADEQUACY OF THE FOREGOING

The preceding sketch defines at best a **piece** of a topological gravity functor. It is defined only for manifolds without boundary, but it behaves correctly under disjoint union: if W_0 and W_1 are two closed four-manifolds, then

$$\sum_{d=d_0+d_1} \mathcal{D}_{d_0}(W_0) \otimes \mathcal{D}_{d_1}(W_1) \mapsto \mathcal{D}_d(W_0 \cup W_1)$$

under the maps of §3.2; this is basically just a definition of the generalized invariant for non-connected manifolds.

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In fact there is reason to think the construction might extend to a larger category. Some years ago, Atiyah [2] proposed a unification of the invariants of Donaldson and Floer, based on a theory of semi-infinite cycles in polarized manifolds. A generalization of Atiyah's cycles which behave naturally under variation of the metric would yield a topological gravity functor for four-manifolds bounded by homology spheres Y , taking values in generalized automorphic forms with coefficients from the Floer homology groups of Y .

Many results which follow from Atiyah's program are known now to be true; but (mostly because of difficulty with compactifications), work on these questions has advanced without using his cycle calculus. I am told, however, that recently there has been progress along the lines he suggested, though in Seiberg-Witten rather than Floer-Donaldson theory. That hope has encouraged me to write this incomplete and probably naive account.

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